

GENERATION AND EVOLUTION OF CAVITATION IN MAGMA UNDER DYNAMIC UNLOADING

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The full system of equations for the problem of rarefaction-wave passage over the magma-melt column in the gravity field is derived with the use of the kinetic theory of phase transformations, and the problem is numerically solved. With allowance for diffusion zones and nucleation frequency as a function of supersaturation, the dependence of the number of cavitation nuclei formed in the course of phase transformations behind the rarefaction-wave front is found. The dynamics of the size distribution of cavitation bubbles along the magma-melt column (1 km) whose viscosity varies dynamically as a function of the concentration of dissolved water is studied.

Key words: *explosive volcanic eruption, dynamic unloading, cavitation.*

Introduction. Volcanic eruptions have versatile forms of manifestation. A typical form for volcanoes is the so-called extrusive eruption [1] in which a red-hot flow of basalt magma is “squeezed out” from the channel. The mechanism of this process is determined by a global decrease in pressure due to magma uprising to the upper levels of the Earth crust and, as a consequence, by liberation of dissolved substances in the form of gas bubbles, which increases the bubble pressure in the magma chamber. It is an excess of this pressure over a certain critical value that leads to extrusive eruption. It was noted [1] that silica-rich magma is noticeably colder, it can be ten orders of magnitude more viscous than basalt magma, and its eruption has an explosive character.

The volcano itself is actually a hydrodynamic system consisting of the magma chamber located in the upper crust at a depth of 7–10 km and filled by high-pressure hot magma [50–70% (wt.) of melted SiO₂], a vertical channel (“conduit”) separated from the chamber by a diaphragm, whose diameter can reach dozens of meters, and a plug closing the channel (diaphragm). The latter is formed in the period between eruptions and consists of hardened magma. Shear motion of the Earth crust caused, in particular, by earthquakes [1] can destroy the plug, which leads to rapid unloading of magma and, as a consequence, to its eruption. This is in line with a typical scheme of a hydrodynamic rarefaction tube with the high-pressure section filled by a liquid, the working sections are both the high-pressure section where the initial stage of cavitation development behind the front of the rarefaction wave propagating over the compressed liquid after diaphragm breakdown is considered and the low-pressure chamber where the process of liquid spreading is studied.

Magma possesses unique physical and chemical properties. The most important of them are the presence of dissolved substances, such as carbon dioxide, sulfur, and water whose concentration can reach 5–7% (wt.), and the high viscosity, which varies from 10² to 10¹² Pa·sec depending on the concentration of the dissolved gas and crystallites contained therein. As it is rather difficult to study real volcanic eruptions because they are rare and unpredictable, simulations of these phenomena within the framework of mechanics of continuous media is very important.

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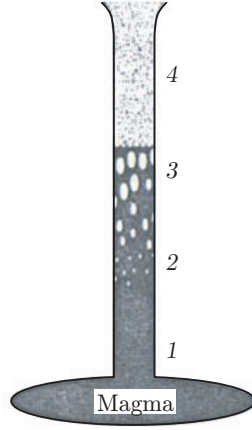


Fig. 1. Development of cavitation in magma under dynamic unloading: 1) uniform flow; 2) bubbly liquid; 3) fragmentation zone; 4) gas suspension.

Among many papers dealing with this issue, let us note the series of investigations [2–6] with an attempt to simulate eruption dynamics in the general formulation. A large class of papers involves consideration of particular processes that accompany this phenomenon. These are papers on growth dynamics of a single bubble in a viscous gas-saturated melt [7–9] and associated thermal effects [10], mechanism of magma solidification under decompression [11, 12], etc.

Nevertheless, despite significant efforts undertaken to study this phenomenon, there are still many issues that require special consideration. In particular, the initial stage of eruption, when cavitation nuclei are generated in magma under rapid unloading, has not been adequately studied. The present paper deals with modeling of this process in heavy magma and studying the growth dynamics of cavitation bubbles in a medium with dynamically varying viscosity due to diffusion of the dissolved gas.

Formulation of the Problem. A vertical column of a gas-saturated magma melt of height H in the gravity field is adjacent to the magma chamber at the bottom and is separated by a diaphragm from the ambient medium on the top (atmospheric pressure is denoted by p_0). We introduce a z axis directed vertically upward with the origin at the column–chamber interface. The initial pressure in magma in the column–chamber system corresponds to the pressure of magma in the chamber with allowance for hydrostatics: $p_i(z) = p_{\text{ch}} - \rho_0 g z$, where ρ_0 is the magma density and p_{ch} is the pressure at $z = 0$.

We assume that the gas dissolved in magma has initially an equilibrium concentration C^{eq} whose dependence on pressure p is determined by Henry’s law. For water dissolved in the magma melt, this dependence has the form [13]

$$C^{\text{eq}}(p) = K_{\text{H}} \sqrt{p}, \quad (1)$$

where K_{H} is Henry’s constant. Correspondingly, the dependence of the initial concentration C_i of the gas dissolved in magma on the z coordinate is determined by the relation $C_i(z) = C^{\text{eq}}(p_i(z))$.

At the initial time ($t = 0$), the diaphragm confining the melt is broken, the surface $z = H$ becomes free, and a rarefaction wave starts propagating vertically downward over the magma. The gas dissolved in magma is supersaturated behind the wave front, which results in spontaneous nucleation and growth of gas bubbles in the melt volume. This process is schematically shown in Fig. 1. The pressure in the magma chamber (at the boundary $z = 0$) is retained constant during the entire process.

To describe the process considered, we write the one-dimensional equations of dynamics of a viscous liquid containing gas bubbles:

— continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial z} = 0; \quad (2)$$

— Navier–Stokes equation

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g + \frac{1}{\rho} \frac{\partial}{\partial z} \left(\mu \frac{\partial v}{\partial z} \right). \quad (3)$$

Here, v , ρ , p , and μ are the mean velocity, density, pressure, and viscosity of the medium, respectively.

Equations (2) and (3) should be supplemented by the equation of state. In the present work, we use the Tait equation [14] where the density of the liquid component is expressed from the equation of state of a two-phase mixture via its mean density and volume concentration of the gas phase k :

$$p = p_0 + \frac{\rho c^2}{n} \left\{ \left(\frac{\rho}{\rho_0(1-k)} \right)^n - 1 \right\} \quad (4)$$

(c is the velocity of sound in the liquid and n is the constant in the Tait equation).

The viscosity of the magma melt as a function of the temperature T and mass concentration C of the dissolved gas is determined as [15, 16]

$$\mu = \mu^* \exp \{ E_\mu(C) / (k_B T) \}, \quad (5)$$

where $E_\mu(C) = E_\mu^*(1 - k_\mu C)$ is the activation energy, E_μ^* is the activation energy for the “dry” melt, k_μ is the empirical coefficient, μ^* is the preexponent, and k_B is the Boltzmann constant. It is seen from Eq. (5) that degassing of the melt increases its viscosity by several orders of magnitude.

The volume concentration k of bubbles formed behind the rarefaction-wave front is determined on the basis of the results obtained by Chernov et al. [17] who showed that the problem of the global kinetics of nucleation and growth of gas bubbles in the melt supersaturated as a result of decompression is similar to the problem of the global kinetics of spontaneous crystallization of a supercooled melt, which was considered by Kolmogorov [18]. Under the condition that nucleation of new centers is possible only in the non-crystallized region, Kolmogorov [18] obtained the time dependences of the fraction of the crystalline mass and the number of crystallization centers for a given growth rate of the crystals and for a nucleation frequency assumed to be uniform over the entire melt volume, which is in line with the approximation of isothermal growth of the crystals. Let us show that the solution obtained in [18] with a certain modification can be applied to the problem considered.

As was noted above, decompression leads to spontaneous nucleation of gas bubbles in the melt. The frequency of their nucleation is determined by the expression

$$J = J^* \exp(-W_*/(k_B T)), \quad (6)$$

where J is the frequency of homogeneous nucleation, $W_* = 16\pi\sigma^3/(3\Delta p^2)$ is the work spent on formation of a critical nucleus in the homogeneous process, $J^* = (2n_g^2 V_g D/d_g)(\sigma/k_B T)^{1/2}$ is the preexponent [8], σ is the surface tension on the melt–gas interface, n_g is the number of potential nucleation centers per unit volume of the melt, which is assumed to be equal to the number of molecules of the gas dissolved in the melt, D is the diffusion coefficient of the gas in the melt, V_g is the volume of the gas molecule, d_g is the mean distance between the neighboring gas molecules in the melt, and $\Delta p = p_s - p$ is the difference between the saturation pressure $p_s(C)$ of the dissolved gas and the current pressure p . The value of Δp can be expressed in terms of melt supersaturation $\Delta C = C - C^{\text{eq}}(p)$ with the use of Henry’s law (1). The nucleation frequency as a function of supersaturation is plotted in Fig. 2.

The dynamics of a single bubble in a viscous liquid is described by the Rayleigh equation with a viscous term [14]

$$R\ddot{R} + (3/2)\dot{R}^2 = \rho^{-1}(p_g - p) + 4\nu R^{-1}\dot{R}, \quad (7)$$

where R is the bubble radius, p_g is the gas pressure in the bubble, p is the ambient pressure, and $\nu = \mu/\rho$ is the kinematic viscosity. Because of the high viscosity of magma melts, the inertial terms in Eq. (7) can be neglected.

The pressure p_g is determined by the diffusion gas flow from the supersaturated melt to the bubble, which is found with the use of the quasi-steady solution of the equation of gas diffusion in the melt

$$C(r) = C_i - (C_i - C^{\text{eq}}(p_g))R/r, \quad (8)$$

where r is the radial coordinate and C_i is the gas concentration far from the bubble. Then, we have

$$\frac{dm_g}{dt} = 4\pi R^2 \rho D \left(\frac{\partial C}{\partial r} \right)_R = 4\pi R \rho D (C_i - C^{\text{eq}}(p_g)), \quad (9)$$

where m_g is the mass of the gas in the bubble.

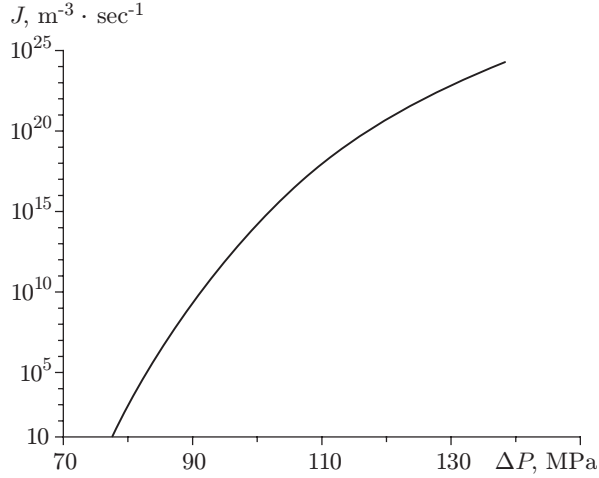


Fig. 2. Nucleation frequency versus supersaturation.

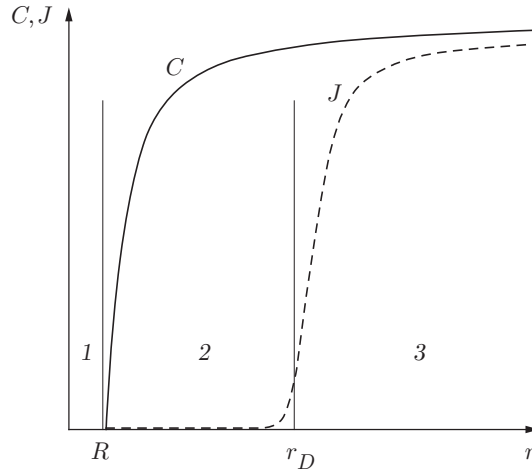


Fig. 3. Schematic of the dependences of the gas concentration (C) and nucleation frequency (J) on the coordinate r : 1) bubble; 2) diffusion layer; 3) nucleation region.

System (7)–(9) should be supplemented by the equation of state of the gas in the bubble (the gas is assumed to be ideal)

$$(4/3)p_g R^3 = (m_g/M)k_B T, \quad (10)$$

where M is the molecular weight of the gas.

It follows from relation (8) that the gas concentration in the melt decreases with approaching the growing bubble, i.e., a diffusion boundary layer is formed around the bubble (Fig. 3). Because of the strong dependence of the nucleation frequency on supersaturation [see Eq. (6) and Fig. 2], we can assume in the first approximation that nucleation of bubbles occurs only outside the diffusion layer (domain 3 in Fig. 3), and the nucleation frequency in this domain can be assumed to be equal to the frequency far from the bubble. Though nucleation of new bubbles inside the diffusion layer is possible, but it does not make any significant contribution to the global gas-release process, because the nucleation frequency in this domain is substantially lower than outside this domain. The diffusion-layer thickness r_D is determined from the condition $J(r_D)/J(r \rightarrow \infty) = 1/10$. Substituting the dependence for the nucleation frequency (6) and taking into account Eq. (8), we obtain

$$r_D = \varkappa R,$$

where $\varkappa = 64\pi\sigma^3/(3k_B T (p_i - p)^2 (1 + \sqrt{p/p_i}) \ln 10)$.

The information given above allows us to modify the solution obtained in [18], as applied to gas-release kinetics. By analogy between the crystallite volume and the volume of the diffusion layer around a single bubble, we have

$$X_D(t) = 1 - \exp\left(-\int_0^t J(\tau) v_D(t-\tau) d\tau\right), \quad (11)$$

where X_D is the total volume of diffusion layers around the bubbles per unit volume of the melt and $v_D = (4\pi/3)(\alpha^3 - 1)R^3$ is the volume of the diffusion layer around a single bubble.

Then, the number of bubbles N_b formed in a unit volume during the time t is

$$N_b(t) = \int_0^t J(\tau) (1 - X_D(\tau)) d\tau. \quad (12)$$

It follows from relations (11) and (12) that bubble nucleation is terminated when the diffusion layers of the growing bubbles completely cover the entire volume ($X_D \rightarrow 1$). According to the estimates [17], the characteristic time of nucleation is significantly smaller than the time of the entire gas-release process, i.e., the main mass of the gas is released at the stage of diffusion growth of the bubbles formed.

Knowing the dependence of the number of bubbles formed in magma on time and their growth rate, we can find the volume fraction κ of bubbles generated in a unit volume of the melt:

$$\kappa = \frac{4\pi}{3} \int_0^t \dot{N}_b(\tau) R^3 (t-\tau) d\tau. \quad (13)$$

Taking into account that the volume of the medium increases in the course of bubble growth, we obtain the relation $k = \kappa/(1 + \kappa)$.

Let us use the following dimensionless variables (marked by the tilde): $\tilde{\rho} = \rho/\rho_0$, $\tilde{\mu} = \mu/\mu_0$, $\tilde{p} = p/p_0$, $\tilde{z} = z/z_0$, $\tilde{v} = v/v_0$, $\tilde{t} = t/t_0$ where $t_0 = z_0/v_0$, $\tilde{R} = R/z_0$, $\tilde{p}_g = p_g/p_0$, $\tilde{m}_g = m_g/m_0$ where $m_0 = (4\pi/3)\rho_0 z_0^3$, and $\tilde{J} = J/J_0$. Here $\mu_0 = \mu^* \exp\{E_\mu(C_i(p_{\text{ch}}))/(k_B T)\}$ is the initial viscosity of magma at $z = 0$, $J_0 = J^* e^{-G}$ is the characteristic nucleation frequency, and $G = 16\pi\sigma^3/(3\rho_{\text{ch}}^2 k_B T)$ is the Gibbs number. Then, system (2)–(13) can be written in dimensionless variables.

The equations of dynamics of the liquid are

$$\begin{aligned} \frac{\partial \tilde{\rho}}{\partial \tilde{t}} + \frac{\partial (\tilde{\rho} \tilde{v})}{\partial \tilde{z}} &= 0, & \frac{\partial \tilde{v}}{\partial \tilde{t}} + \tilde{v} \frac{\partial \tilde{v}}{\partial \tilde{z}} &= -\frac{\text{Eu}}{\tilde{\rho}} \frac{\partial \tilde{p}}{\partial \tilde{z}} - \frac{1}{\text{Fr}} + \frac{1}{\text{Re}} \frac{1}{\tilde{\rho}} \frac{\partial}{\partial \tilde{z}} \left(\tilde{\mu} \frac{\partial \tilde{v}}{\partial \tilde{z}} \right), \\ \tilde{\mu} &= \exp \frac{E_\mu^* k_\mu (C_i(p_{\text{ch}}) - C)}{k_B T}, & \tilde{p} &= 1 + \frac{\tilde{c}^2}{n} \left\{ \left(\frac{\tilde{\rho}}{1-k} \right)^n - 1 \right\}, \end{aligned}$$

where $\text{Eu} = p_0/(\rho_0 v_0^2)$ is the Euler number, $\text{Fr} = v_0/(gt_0)$ is the Froude number, $\text{Re} = z_0 v_0/\nu_0$ is the Reynolds number, and $\nu_0 = \mu_0/\rho_0$. If we assume that $v_0 = (p_0/\rho_0)^{1/2}$, $z_0 = p_0/(\rho_0 g)$, and $t_0 = (1/g)(p_0/\rho_0)^{1/2}$, the Euler and Froude numbers become equal to unity, and the Reynolds number takes the form $\text{Re} = (p_0/\rho_0)^{3/2}/(\nu_0 g)$.

The equations of bubble nucleation and growth kinetics are

$$k = \kappa/(1 + \kappa),$$

$$\kappa = \frac{4\pi}{3} J_0 z_0^3 t_0 \int_0^{\tilde{t}} \tilde{J}(\tilde{\tau}) \tilde{R}^3 (\tilde{t} - \tilde{\tau}) \exp \left\{ -\frac{4\pi}{3} J_0 z_0^3 t_0 (\alpha^3 - 1) \int_0^{\tilde{\tau}} \tilde{J}(\tilde{\tau}') \tilde{R}^3 (\tilde{\tau} - \tilde{\tau}') d\tilde{\tau}' \right\} d\tilde{\tau},$$

$$\tilde{J} = \exp \{-G[(\tilde{p}_{\text{ch}}/\Delta\tilde{p})^2 - 1]\}, \quad \tilde{R} \ddot{\tilde{R}} + \frac{3}{2} \dot{\tilde{R}}^2 = \frac{\text{Eu}}{\tilde{\rho}} (\tilde{p}_g - \tilde{p}) + \frac{4\tilde{v}}{\text{Re}} \frac{\dot{\tilde{R}}}{\tilde{R}},$$

$$\frac{1}{3} \text{Re Pr}_D \frac{d\tilde{m}_g}{dt} = \tilde{R} (C_i - C^{\text{eq}}(\tilde{p}_g)), \quad \tilde{p}_g \tilde{R}^3 = (\rho_0/p_0) (\tilde{m}_g/M) k_B T.$$

Here $\text{Pr}_D \equiv \text{Sh} = \nu_0/D$ is the Prandtl (Sherwood) number; as in the momentum equation, $\text{Eu} = 1$ in the Rayleigh equation.

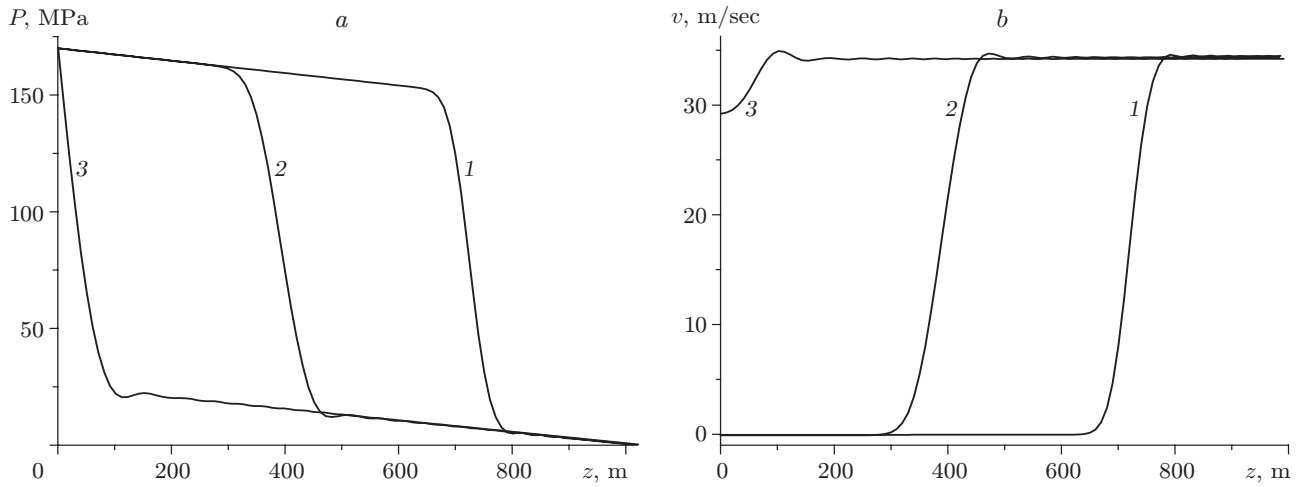


Fig. 4. Pressure (a) and velocity of magma (b) as functions of z for $t = 0.21$ (1), 0.42 (2), and 0.67 sec (3).

The constructed system of equations completely determines the dynamic of rarefaction-wave passage over the magma-melt column in the gravity field and allows finding the dependence of the number of cavitation nuclei formed in the course of phase transformations behind the rarefaction-wave front and the size distribution of cavitation bubbles over the entire length of the magma-melt column.

Calculation Results. The calculations were performed for the following problem parameters typical of explosive volcanic eruptions: magma-column height $H = 1$ km, pressure in the volcano center located at a depth of approximately 5–7 km, $p_{\text{ch}} = 1700$ atm, and temperature $T = 1150$ K, which corresponds to the melting point of magma at a pressure $p = p_{\text{ch}}$ [12]. The following values of magma properties are used in the calculations: $\rho_0 = 2300$ kg/m³, $K_{\text{H}} = 4.33 \cdot 10^{-6}$ Pa^{-1/2}, $D = 2 \cdot 10^{-11}$ m²/sec, $\sigma = 0.076$ J/m², $E_{\mu}^* = 5.1 \cdot 10^{-19}$ J, $k_{\mu} = 11$, and $\mu^* = 10^{-2.5}$ Pa·sec. Correspondingly, the numbers of similarity in the problem have the following order: $\text{Re} \approx 1$, $\text{Pr}_D \approx 10^{11}$, and $G = 16$. For simplicity, all calculation results are presented in dimensional variables.

Figure 4 shows the pressure fields and the distributions of the liquid velocity over the column height for three different time instants. It is seen from Fig. 4b that the liquid is accelerated to a velocity $v \approx 35$ m/sec under the action of pressure behind the rarefaction-wave front, which agrees with the estimate that can be derived from simple considerations: $p_{\text{ch}}/(\rho_0 c) \approx 30$ m/sec. At the time described by curve 3, the rarefaction wave reaches the bottom of the column (coordinate $z = 0$), is reflected from the bottom, and is transformed to a compression wave. If the wave passes over the column many times, the liquid velocity increases.

Figure 5 shows the number of nucleation centers formed in the process as a function of z . The function has a stepwise shape because the characteristic nucleation time is much smaller than the characteristic time of the entire process [17]. This is seen from Fig. 6, which shows the time evolution of the bubble-nucleation rate in a narrow time interval for $z = 0$. Note, the numbers of bubbles obtained in the present work are slightly smaller than those determined in [17]. The reason is that decompression was assumed to be instantaneous in [17], whereas the pressure wave in the present problem has a gently sloping front (see Fig. 4a), and nucleation of bubbles occurs at lower values of supersaturation and, hence, at lower frequencies. In addition, the bubble-growth mechanism [17] ignored the influence of viscosity manifested at the initial stage of growth and restraining the latter, which was taken into account in the present work.

The model presented allows calculation of the size distribution of the bubbles along the column. The distribution function in Fig. 7 is constructed for three different times. Two stages of bubble growth can be distinguished: the initial stage when the bubble growth is restrained by viscous tension forces and the diffusion stage when the bubble-growth dynamics is determined only by the diffusion gas flow from the supersaturated melt to the bubble, which is described by the root dependence of the bubble size on time [7].

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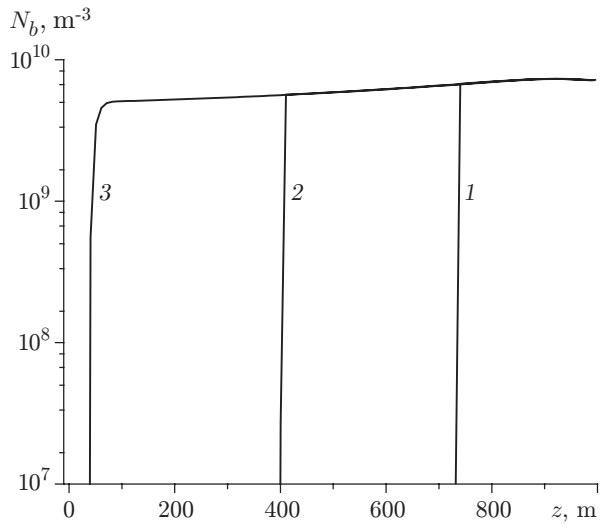


Fig. 5

Fig. 5. Number of bubbles versus z for $t = 0.21$ (1), 0.42 (2), and 0.67 sec (3).

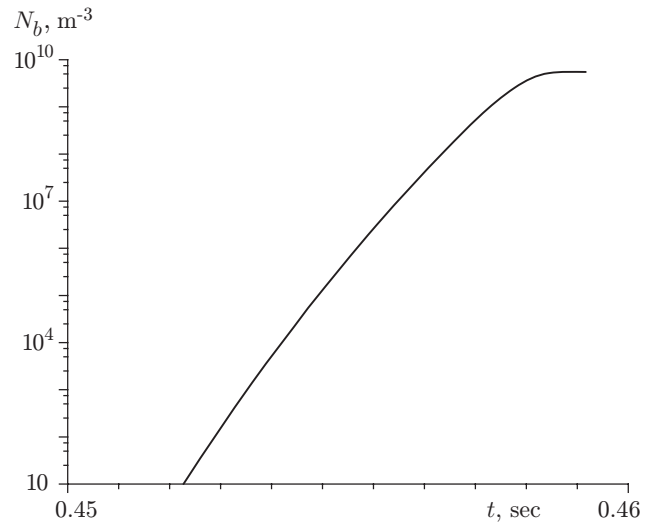


Fig. 6

Fig. 6. Number of bubbles as a function of time for $z = 0$.

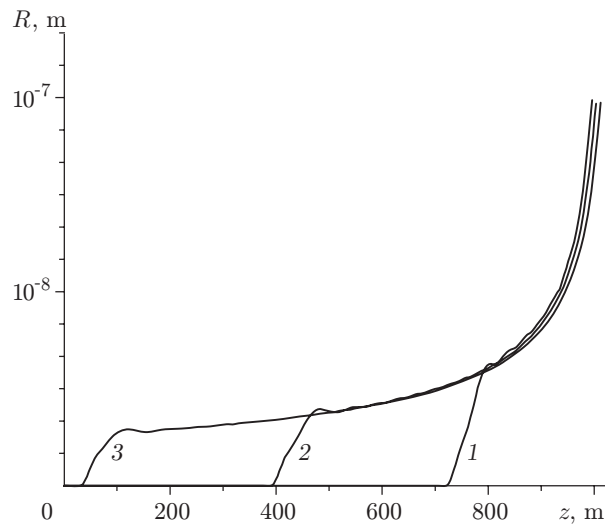


Fig. 7. Size distribution of bubbles along the column for $t = 0.21$ (1), 0.42 (2), and 0.67 sec (3).

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